Solutions to JEE Advanced Home Practice Test -6 |JEE 2024 | Paper-2

PHYSICS

1.(6) Apparent depth $=\dfrac{2R+x}{\mu}$

For refraction at curved surface

$$u=-\left(R+\frac{2R+x}{\mu}\right)$$

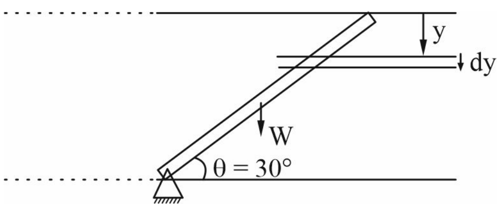
$$v=-(R+x) \quad \Rightarrow \quad \frac{\mu}{-(R+x)}-\frac{1}{-\left(R+\frac{2R+x}{\mu}\right)}=\frac{\mu-1}{-R}$$

$\Rightarrow$ We should get: $x^2+36x-256=0$ $\Rightarrow$ $x=6cm$

2.(8) For rod to be in equilibrium

$\tau_W=\tau_B$ **. . . (i)**

$$\tau_W=\rho ALg\frac{L}{2}\cos 30°$$



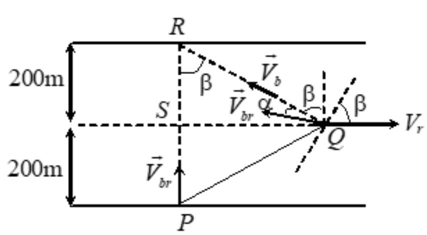
$$\tau_B=\int_0^6 \rho_0\left(1+\frac{y}{6}\right)A\frac{dy}{\sin 30°}g\,(12-2y)\cos 30°=96\rho_0 Ag\cos 30°$$

Putting the values in equation (i), use get

$$\rho=\frac{4\rho_0}{3}$$

3.(3) To reach point *R* from *Q*, the boat should be aimed at point *R* as seen from ground

$$\vec{V}_r\cos\beta=\vec{V}_{br}\sin\alpha$$



$$2\times\frac{2}{\sqrt{5}}=4\sin\alpha$$

$$\sin\alpha=\frac{1}{\sqrt{5}}$$

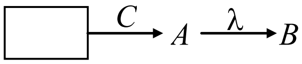
$\Rightarrow$ $\tan\alpha=\dfrac{1}{2}$

Also $\tan\beta=\dfrac{1}{2}$

$\theta=\alpha+\beta$

$\Rightarrow$ $\tan\theta=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\dfrac{4}{3}$ $\therefore$ $n=3$

4.(1) $\frac{dN_A}{dt} = +C - \lambda N_A$; $\int_0^{N_A} \frac{dN_A}{C - \lambda N_A} = \int_0^t dt$

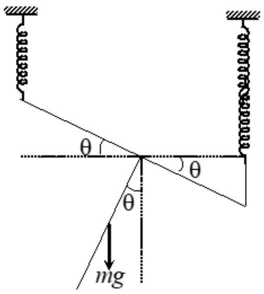


$\Rightarrow \ln \frac{C - \lambda N_A}{C} = -\lambda t$; $N_A = \frac{C}{\lambda}(1 - e^{-\lambda t})$

$N_B =$ no. of N_A decayed $= Ct - \frac{C}{\lambda}(1 - e^{-\lambda t})$

at $t = \frac{1}{\lambda} \Rightarrow N_B = \frac{C}{\lambda} - \frac{C}{\lambda} + \frac{Ce^{-1}}{\lambda} = \frac{C}{\lambda}e^{-1} \Rightarrow \frac{100 \times 10^6}{37} \times 0.37 = 1 \times 10^6$ $\therefore$ $x = 1$

5.(3) Let the body be displaced through an angle θ about its mean position.
Net torque on the body at this position



$\tau = -\left(2Ka\theta a \cos\theta + mga \sin\theta\right)$

$= -\left(2Ka^2 + mga\right)\theta$

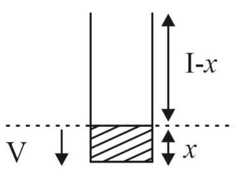
(where θ is small, sin θ = θ and cos θ = 1)

Angular acceleration $\alpha = \frac{\tau}{I} = -\frac{-\left(2Ka^2 + mga\right)\theta}{\frac{M \times (2a)^2}{12} + \frac{M(2a)^2}{3}}$

or $\alpha = -\frac{\left(2Ka^2 + mga\right)}{\frac{5ma^2}{3}}$

$\Rightarrow$ $\omega = \sqrt{\frac{\left(2Ka^2 + mga\right)}{5ma^2/3}} = \sqrt{\left(\frac{6K}{5m} + \frac{3g}{5a}\right)} = \sqrt{\frac{6}{5} \times \frac{96}{6} + \frac{30}{5 \times 5} \times 104} = 12$ rad/s; $n = 3$

6.(5) $c_{air} = \frac{\epsilon_0 (1 - x)}{d}$; $c_{liq} = k\frac{\epsilon_0 (x)}{d}$



$c = c_{air} + c_{liq} = \frac{\epsilon_0}{d}[1 - x + kx]$

$Q = cV = \frac{V \epsilon_0}{d}[1 + (k-1)x]$

$I = \frac{dQ}{dt} = \frac{V \epsilon_0}{d}[0 + (k-1)v] = \frac{500 \times 8.85 \times 10^{-12}}{8.85 \times 10^{-3}}[10 \times 10^{-3}] = 5 \times 10^{-9} A$

7.(ACD) $u = -30\,cm$; $v = ?$

$f = -20\,cm$; $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ $\Rightarrow v = -60$

$|m| = \left|-\frac{v}{u}\right| = |-2| = 2$; $v_{ox} = \sqrt{5}\cos(\tan^{-1} 2) = 1\,m/s = \hat{i}$

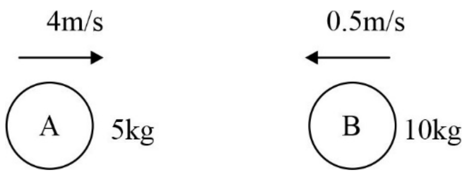
$v_{oy} = \sqrt{5}\sin(\tan^{-1} 2) = 2\,m/s = +2\hat{j}$

$\frac{v_{Ix}}{v_{ox}} = -m^2 = -4 \Rightarrow v_{Ix} = -4\,m/s = -4\hat{i}$

$\frac{v_{Iy}}{v_{oy}} = m = -2 \Rightarrow v_{Iy} = -4 = -4\hat{j}$; $\quad v_I = \sqrt{v_{Iy}^2 + v_{Ix}^2} = 4\sqrt{2}\, m/s$

$v_{I/o} = (-5\hat{i} - 6\hat{j})\, m/s$, speed = $\sqrt{5^2 + 6^2} = \sqrt{61}\, m/s$

8.(AB) **(A)** $M_A = 5kg.$

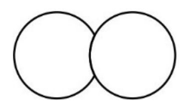


Deformation will take place until both of them achieve common velocity. $\because$ $F_{ext} = 0$

P Momentum will be conserved.

$P_i = P_f$ (till they achieve common velocity)

$5 \times 4 - 10 \times 0.5 = (5 + 10) V_c$



$20 - 5 = 15 V_c$; $\boxed{V_c = \frac{15}{15} = 1 m/s}$

Impulse $\rightarrow$ Area of $F - t$ graph.

For deformation.

$\int F \cdot dt = \Delta P \cdot$ (of any particle.) $\quad -\frac{1}{2} \times 150 \times t = (5 \times 1 - 20 \cdot)$

$\frac{1}{2} \times 150 \times t = 15$; $\boxed{t = \frac{2}{10} = 0.2 \sec}$

(B) $e = \frac{Impulse\ of\ Reformation}{Impulse\ of\ deformation} = \frac{Area\ from\ t = 0.20\ to\ t = 0.30 \sec}{Area\ from\ t = 0\ \ to\ t = 0.20 \sec}$

$= \frac{\frac{1}{2} \times (0.3 - 0.2) \times 150}{\frac{1}{2} \times (0.2 - 0) \times 150}$; $\quad e = \frac{0.1}{0.2} = 0.5$

For final velocities; $\quad e = \frac{v_2 - v_1}{u_1 - u_2}$ $\quad$ and $\quad m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$5 \times 4 + 10(-0.5) = 5 v_1 + 10 v_2$; $\quad 15 = 5 v_1 + 10 v_2$; $\quad \boxed{3 = v_1 + 2 v_2}$

$e = \frac{v_2 - v_1}{u_1 - u_2}$; $\quad 0.5 = \frac{v_2 - v_1}{4 - (-0.5)}$

Solving both

$v_1 + 2 v_2 = 3$; $\quad \frac{v_2 - v_1 = 0.5 \times 4.5}{3 v_2 = 3 + 2.25}$

(1) $\quad 3 v_2 = 5.25$; $\quad v_2 = 1.75\, m/s$

$v_1 = 1.75 - 2.25 = -0.5\, m/s$

9.(ACD) $mv_0 y = mvy + \frac{ml^2}{3}\omega$

$\Rightarrow \quad v_0 y = vy + \frac{\omega l^2}{3} \quad \ldots.(i)$

$ev_0 = \omega y - v \quad \ldots.(ii)$

Solving (i) and (ii) we can calculate the value of v and ω for different cases.

For $y = \frac{l}{3}$ and $e = 1$, we get $v = \frac{-v_0}{2}$ and $\omega = \frac{3v_0}{2\ell}$

For $y = \frac{l}{2}$ and $e = 1$, we get $v = \frac{-v_0}{7}$ and $\omega = \frac{12V_0}{7\ell}$

For $v = 0$, we get $y = \sqrt{\frac{e}{3}}\ell$

For $mv_0 = mv + \frac{m\omega l}{2}$, we get $y = \frac{2\ell}{3}$

10.(ABD) $E = -\frac{GMm}{2r}$

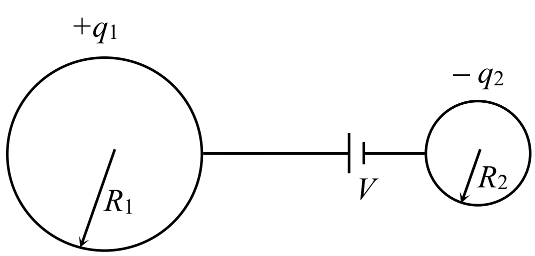
The drag force causes energy loss at the rate

$$-Fv = \alpha v v = \alpha v^2 = \frac{\alpha GM}{r} \qquad \left(v = \sqrt{\frac{GM}{r}}\right)$$

$$-\frac{dE}{dt} = -\frac{GMm\frac{dr}{dt}}{2r^2} = \frac{\alpha GM}{r}$$

Integrating, we get $r = r_0 e^{-\frac{2\alpha t}{m}}$, where we have used $r = r_0$ at $t = 0$.

11.(ABC) $\frac{q_1}{4\pi\varepsilon_0 R_1} - \left(-\frac{q_2}{4\pi\varepsilon_0 R_2}\right) = V$



Also number of electrons emitted = $\frac{q_1 - q_2}{e}$

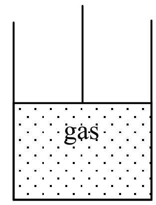
12.(BCD) At $t = 1s$; $\quad \vec{v} = 2\hat{i} + 2\hat{j}$

$\vec{a} = 2\hat{i} + 2\hat{j}$; $\quad a_t = \vec{a}.\hat{v} = \frac{6}{\sqrt{5}} m/s^2$

$a_r = \sqrt{a^2 - a_t^2} = \frac{2}{\sqrt{5}} m/s^2$; $\quad R = \frac{v^2}{a_r} = \frac{5\sqrt{5}}{2} m$

13.(750) $W_g + W_A + W_{ext} = 0$

$W_{ex} = -[W_g + W_A]$

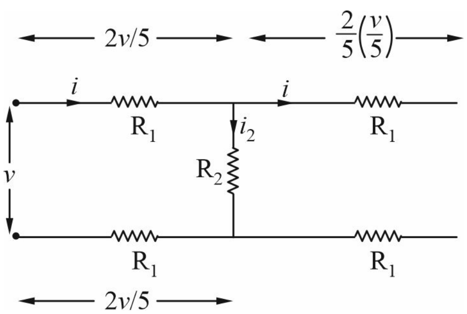


$W_g = nRT \ln\left(\frac{V_2}{V_1}\right) = 1750J$

$W_A = -P_0(V_2 - V_1) = \frac{nRT}{(V_2 - V_1)}(V_2 - V_1) = -nRT$

$= 1 \times \frac{25}{3} \times 300 = -2500J$ $\therefore$ $W_{ext} = 750$ J

14.(1.60) $i = i_1 + i_2$



$\frac{2v/5}{R_1} = \frac{v/5}{R_2} + \frac{2v/25}{R_1}$ $\Rightarrow$ $\frac{R_1}{R_2} = 1.6$

15.(5) $\because$ $\frac{\sigma}{\sigma - \delta} = \frac{10.0 \pm 0.1}{5.0 \pm 0.1}$ $\Rightarrow$ $(5.0 \pm 0.1)\sigma = (10.0 \pm 0.1)\sigma - (10.0 \pm 0.1)s$

$\Rightarrow$ $(10.0 \pm 0.1)s = (5.0 \pm 0.2)\sigma$ $\Rightarrow$ $r = \frac{\sigma}{s} = \frac{10.0 \pm 0.1}{5.0 \pm 0.2} = \frac{10.0 \pm 1\%}{5.0 \pm 4\%} = 2.0 \pm 5\%$

16.(336) When reservoir is lowered by x, let the level of water fall by y

$x - \frac{y}{6} = y$ $\therefore$ $x = \frac{7y}{6}$ $\Rightarrow$ $y = \frac{6x}{7}$

For $x = 21$ cm, $y_1 = 18$ cm

For $x = 21 + 49 = 70\ cm$; $y_2 = 60\ cm$

$\therefore$ $\frac{\lambda}{4} + e = 18$

$\frac{3\lambda}{4} + e = 60$ ....(ii)

(ii) - (i) gives

$\frac{\lambda}{2} = 42 \Rightarrow \lambda = 84\ cm = 0.84\ m$ $\therefore V = \lambda f = 0.84 \times 400 = 336\ ms^{-1}$

17.(0.69) Rate of increase of energy in inductor is maximum when $e^{-(R/L)t} = \frac{1}{2}$

18.(2) $12 = k(45 - 15)$ $\Rightarrow$ $k = 0.4$

Rate of loss of heat $= k(20 - 15) = 2W$

CHEMISTRY

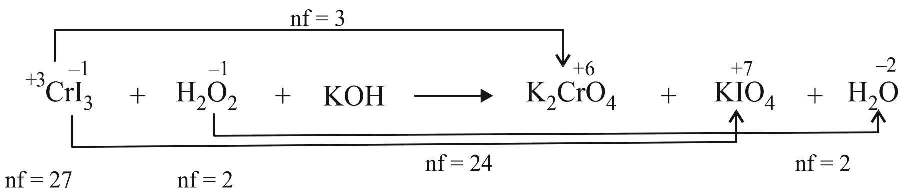
1.(13) For element X, the maximum difference in I.E is in between IE_3 and IE_4; hence, after removal of 3rd electron element must have achieve Noble gas configuration. So, it belongs to group 13.

2.(2) KO_2 and O_2 are paramagnetic in nature have 1 and 2 unpaired e^- respectively.

Electronic configuration of O_2 (more than $14e^-$) is:

$$\sigma_{1s}^2, \overset{*}{\sigma}_{1s}^2, \sigma_{2s}^2, \overset{*}{\sigma}_{2s}^2, \sigma_{2pz}^2 \begin{vmatrix} \pi_{2px}^2 \\ \pi_{2py}^2 \end{vmatrix}\begin{vmatrix} \pi^*_{2px}{}^1 \\ \pi^*_{2py}{}^1 \end{vmatrix}$$

3.(6) The balanced equation is



$$2CrI_3 + 27H_2O_2 + 10KOH \longrightarrow 2K_2CrO_4 + 6KIO_4 + 32H_2O$$

4.(2) $\underset{\text{Pyrolusite}}{MnO_2} \xrightarrow[KNO_3]{KOH,\Delta} \underset{\text{green melt}}{MnO_4^{2-}} \xrightarrow{\text{Alkali}+H_2O} \underset{\text{green solution}}{MnO_4^{2-}}$

$$3K_2MnO_4 + 2H_2O \rightleftharpoons \underset{\text{purple}}{2KMnO_4} + \underset{\text{black}}{MnO_2}\downarrow + 4KOH$$

$$2KMnO_4 \xrightarrow{200°C} \underset{\text{green}}{K_2MnO_4} + \underset{\text{black}}{MnO_2} + O_2$$

Oxidation number of Mn in K_2MnO_2 is +6

Oxidation number of Mn in MnO_2 is +4

Difference = 2

5.(0) At pH = 8, the charge on the respective groups will be:

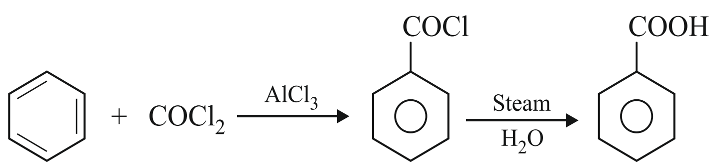
pK_a	9.82	3.86	10.07	10.53	2.18
Charge	+1	–1	0	+1	–1

Net charge = 0

6.(5) $C_9H_{12}O$ has DBE = $\frac{20-12}{2} = 4$

$C_9H_{12}O$ must have a benzene ring.

Given



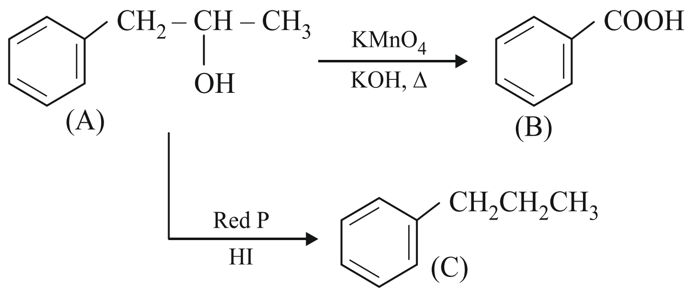
(B)

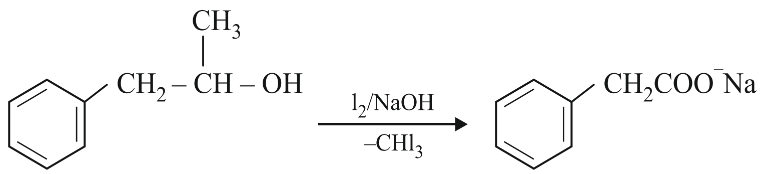
→ A does react with Na, hence must have a phenolic or alcoholic group or carboxylic group.

→ A gives positive iodoform test, hence must have ketone or $-\underset{\text{OH}}{\underset{|}{\text{CH}}}-\text{CH}_3$ group

Since it gives the Lucas test in 5 min.

→ Hence, a must be a secondary alcohol.





Molecular weight of C = 120 g/mol

$X = 120 \qquad \frac{X}{24} = 5$

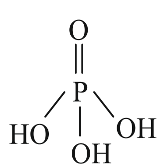
7.(CD)

Compound	**EAN**	
$Ni(CO)_4$	36	
$[Fe(CO)_5]$	36	
$[Fe(NH_3)_6]^{2+}$	36	
$[Fe(H_2O)_5NO]^{2+}$	37	(NO^+ is ligand)
$[Mn(CO)_6]$	37	
$[Ti(CO)_6]$	34	
$[Mn(C_2O_4)_3]^{3-}$	34	
$[Cr(OX)_3]^{3-}$	33	

8.(BD) For figure 1, rate of formation of B is higher than rate of formation of C.
For figure 2, rate of formation of C is greater than rate of formation of B, hence $k_2 > k_1$

9.(AC) Oxygen liberated at anode which is made up of graphite, corrodes the anode forming CO and CO_2.
Surface becomes rough and radiation loss of heat is also prevented.

10.(ACD)

(A) Structure of H_3PO_4 is 

(B) Electronic configuration of Zn is $[Ar]3d^{10}4s^2$

(C) B_2H_6 has two $3C-2e^-$ bond and four $2C-2e^-$ bonds

(D) Stability of $Pb^{2+} > Pb^{4+}$ while $Sn^{4+} > Sn^{2+}$ hence Pb^{4+} is an O.A. and Sn^{2+} is a R.A.

11.(ABD)

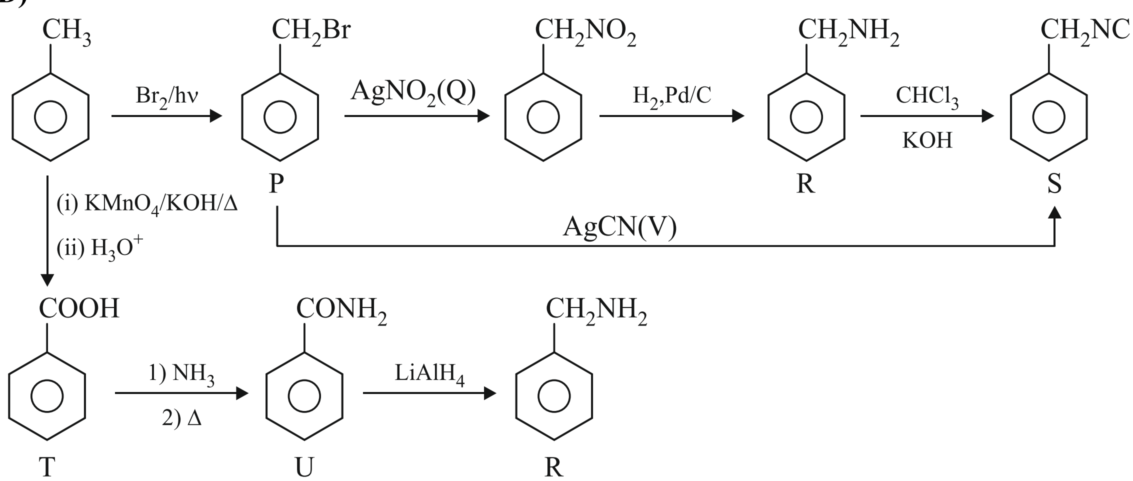
(A) Electron withdrawing group favours nucleophilic addition

(B) Good leaving group favours acyl S_N2 reaction

(C) Smaller acids are more reactive

(D) Smaller alkyl groups in ester favours hydrolysis

12.(BCD)



13.(3) Equivalence point 1 of H_2S is at $(0.1\times 40) = 0.08\times V_1$

$V_1 = 50ml$

At equivalence point 2 of H_2S

$0.1\times 2\times 40 = 0.08\times V_2$

$V_2 = 100ml$

At 40 ml

$$\begin{array}{llll} H_2S & + \; NaOH & \longrightarrow NaHS + H_2O \\ 4m\,mol & 3.2m\,mol & \\ 0.8\,m\,mol & - & 3.2\,m\,mol \end{array}$$

$$pH = 7 + \log\frac{3.2}{0.8} = 7.6$$

At 50 ml

$$\begin{array}{llll} H_2S & + \; NaOH & \longrightarrow NaHS & + \; H_2O \\ 4m\,mol & 4m\,mol & & \\ - & - & 4m\,mol & \end{array}$$

$$pH = \frac{pK_{a_1} + pK_{a_2}}{2} = \frac{7+14.2}{2} = 10.6$$

$\Delta pH = 10.6 - 7.6 = 3$

14.(8) $\Delta T_f = 0°C - (-6°C) = 6°C$

$\Delta T_f = i K_f m$

$i = 1$

$6 = 1 \times 1.86 \times \dfrac{X}{62 \times 4}$

$X = 800g \quad = 8 \times 10^2 g$

Y = 8

15.(2) At node $\psi_{2s}^2 = 0$

$\left(x^2 - \dfrac{r}{r_0}\right) = 0$

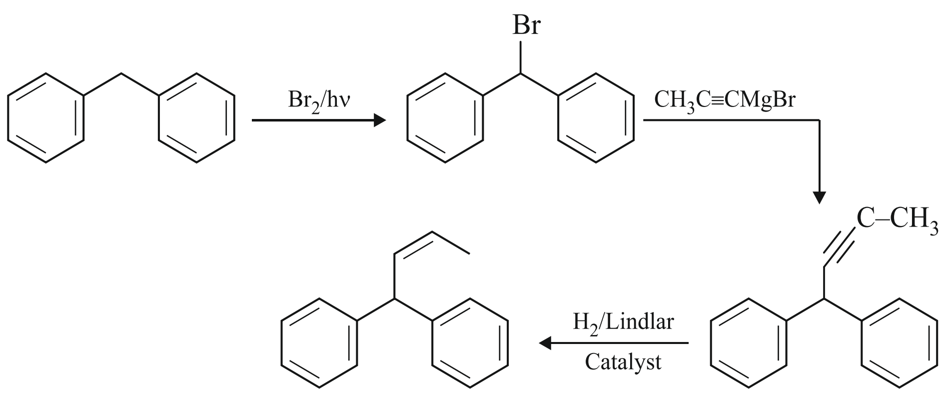
$x^2 = \dfrac{r}{r_0} \qquad r = 4r_0$

$x^2 = 4 \qquad x = 2$

16.(13.94)

Molecular mass of X = 168g/mol

Moles of $X = \dfrac{16.8g}{168g} = 0.1$ mole



Molar mass of Y = 208 g/mol

Expected moles of Y = 0.1 mole (100% yield)

Yield of Y = 0.067 moles

$W_y = 0.067 \times 208 = 13.936 \approx 13.94$

17.(46.08) $H_2 + I_2 \rightleftharpoons 2HI$

$i = \dfrac{10}{400R} \quad \dfrac{10}{400R} \quad 0 \quad$ (Consider $\dfrac{10}{400R} = n$)

f : $n - x \quad n - x \quad 2x$

$$K_C = \frac{(2x/10)^2}{\left(\frac{n-x}{10}\right)^2} = 9$$

$\Rightarrow \dfrac{2x}{(n-x)} = 3 \qquad \Rightarrow 2x = 3n - 3x$

$\Rightarrow 5x = 3n \qquad \Rightarrow x = \dfrac{3n}{5} = \dfrac{3}{5}\left[\dfrac{120}{400}\right] = \dfrac{36}{200}$

$\therefore W_{HI} = 2\left[\dfrac{36}{200}\right][128] = 46.08$

18.(1.9) FeS is much more soluble than HgS in acidic medium. We want H^+ large enough to prevent FeS to precipitate but HgS to:

$K_a = \dfrac{[H^+]^2[S^{2-}]}{[H_2S]}; \quad K_{sp}(FeS) = [Fe^{2+}][S^{2-}]$

$\dfrac{K_a}{K_{sp}} = \dfrac{10^{-21}}{6\times10^{-19}} = \dfrac{[H^+]^2}{[H_2S][Fe^{2+}]}$

$[H^+]^2 = \dfrac{10^{-2}\times10^{-1}\times10^{-2}}{6} = \dfrac{10^{-4}}{0.6} = 1.667\times10^{-4}, [H^+] = 1.291\times10^{-2}$

$pH = 2 - \log 1.291 \qquad = 2 - 0.1 = 1.9$

MATHEMATICS

1.(1) $2\arg(z^{1/3}) = \arg(z^2 + \bar{z}z^{1/3})$

$\Rightarrow \quad \arg(z^{2/3}) = \arg(z^2 + \bar{z}z^{1/3}) \Rightarrow \arg(z^2 + \bar{z}z^{1/3}) - \arg(z^{2/3}) = 0$

$\Rightarrow \quad \arg\left(\dfrac{z^2 + \bar{z}z^{1/3}}{z^{2/3}}\right) = 0 \;\Rightarrow\; \arg\left(z^{4/3} + \dfrac{\bar{z}}{z^{1/3}}\right) = 0$

$\Rightarrow \quad z^{4/3} + \dfrac{\bar{z}}{z^{1/3}} = \bar{z}^{4/3} + \dfrac{z}{\bar{z}^{1/3}}$ (because $\arg(z) = 0 \Rightarrow z = \bar{z}$)

$\Rightarrow \quad \bar{z}^{1/3}(z^{5/3} + \bar{z}) = z^{1/3}(\bar{z}^{5/3} + z) \Rightarrow \bar{z}^{1/3}z^{1/3}z^{4/3} + \bar{z}^{4/3} = z^{1/3}\bar{z}^{1/3}\bar{z}^{4/3} + z^{4/3}$

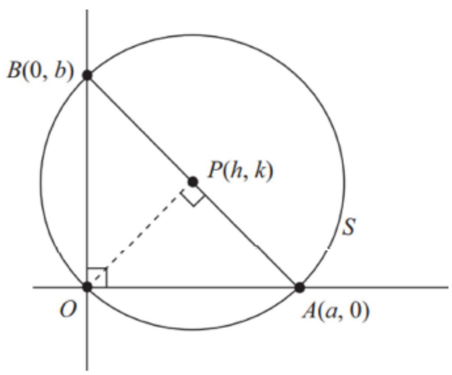
$\Rightarrow \quad |z|^{2/3} z^{4/3} + \bar{z}^{4/3} = |z|^{2/3}\bar{z}^{4/3} + z^{4/3} \Rightarrow z^{4/3}(1 - |z|^{2/3}) - \bar{z}^{4/3}(1 - |z|^{2/3}) = 0$

$\Rightarrow \quad (z^{4/3} - \bar{z}^{4/3})(1 - |z|^{2/3}) = 0$

Since z is a non-real complex number, $z \neq \bar{z}$, and so $z^{4/3} \neq \bar{z}^{4/3}$

Therefore, $|z|^{2/3} = 1 \Rightarrow |z| = 1$

2.(2) Let the co-ordinates of A and B be $(a, 0)$ and $(0, b)$ respectively, so that the equation to the variable circle becomes $x^2 + y^2 - ax - by = 0$



The equation for S is $x^2 + y^2 - ax - by = 0$

Note that since $\angle AOB = \dfrac{\pi}{2}$,

AB is a diameter of the circle

We have $a^2 + b^2 = 16r^2$ ………(1)

Let the foot of perpendicular P have the co-ordinates (h, k). Since $OP \perp AB$, we obtain $m_{AB} = -\dfrac{h}{k}$

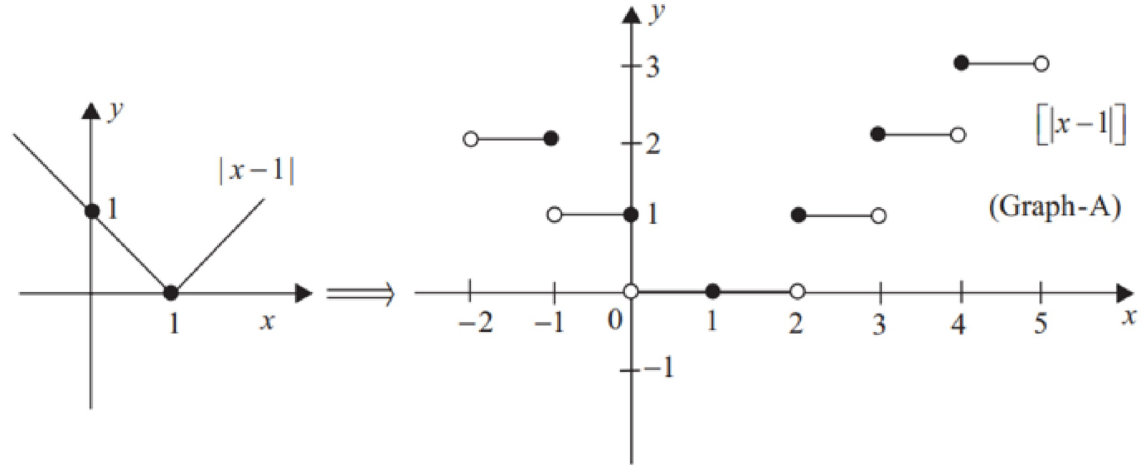
Equation of $AB : \dfrac{x}{a} + \dfrac{y}{b} - 1 = 0$ ……….(2)

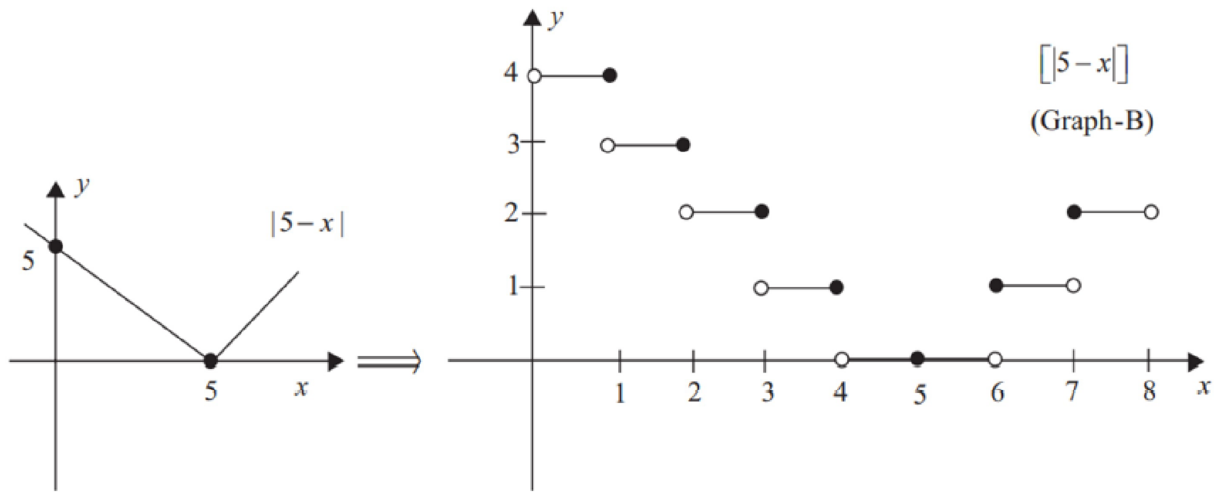
Also, Equation of $AB : y - k = -\dfrac{h}{k}(x - h)$ ……….(3)

(2) & (3)

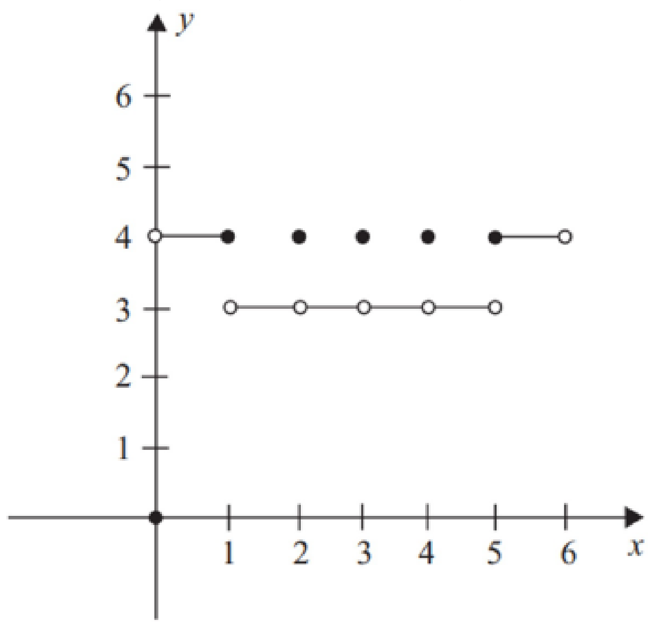
$\Rightarrow \quad a = \dfrac{h^2 + k^2}{h},\ b = \dfrac{h^2 + k^2}{k} \quad \Rightarrow (x^2 + y^2)^3 = 16(rxy)^2 \Rightarrow \lambda = 16$

3.(5)





Now add the graphs of A and B point by point:



We see that the value of (graph A + graph B) is 4 for the following values of x:

x : (0, 1], {2, 3, 4}, [5, 6).

Hence, $D = R - \{(0, 1], 2, 3, 4, [5, 6)\}$. We see that there are 5 integers which do not lie in the domain of the given function, namely 1, 2, 3, 4, 5. Therefore, the correct answer is 5

4.(3) The general, rth term in the series is $t_r = (r+2)\left(\sum_{k=1}^{n-r+1} k\right)$ which equals $(r+2)\cdot\frac{(n-r+1)(n-r+2)}{2}$. Thus,

$$\sum_{r=1}^{n} t_r = \sum_{r=1}^{n} \frac{(r+2)(n-r+1)(n-r+2)}{2}$$

The highest degree term in this summation is $\frac{n^4}{24}$. Thus, the limit is $\frac{72}{n^4}\times\frac{n^4}{24} = 3$.

5.(1) Firstly, note that to calculate the total number of matrices in the sample space, we may place the three 1's in any of the 9 entries of M and the remaining 6 entries would be all 0.

Hence, total number of matrices M in the sample space is ${}^9C_3 = 84 = \lambda$

For M to be non-singular, each row must have exactly one 1 and no two 1's must be present on the same column. This can be done in 6 ways. Hence, probability is $\frac{6}{84} = \frac{1}{14} \Rightarrow t = 14$

For trace (M) = 0, 0's are present on the principal diagonal. Hence, 1's can be placed on any of the 6 remaining entries. Hence, probability is $\frac{5}{21} \Rightarrow s = 5$ $\quad\quad \therefore \frac{\lambda}{t} - s = 1$

6.(3) $(EM)^T = 20I$

Take transpose on both sides

$$EM = 20I \quad \text{..........(1)}$$

$$(E+M)^T = 17(E-M)^T$$

$$E^T + M^T = 17(E^T - M^T)$$

$$16E^T = 18M^T$$

Take transpose on both sides

$$16E = 18M \quad \text{.........(2)}$$

From Equations (1) and (2), we get

$$E = \pm\frac{3\sqrt{10}}{2}I\ ;\ M = \pm\frac{4\sqrt{10}}{3}I$$

$$E^2 + M^2 = \frac{725}{18}I \Rightarrow a+b = 743 \Rightarrow a+b-740 = 3$$

7.(AC) Both the lines have been specified in non-parametric form, which we can easily convert to parametric form:

$\vec{r}\times\vec{a} = \vec{b}\times\vec{a} \Rightarrow \vec{r} = \vec{b} + \lambda\vec{a}$ where $\lambda \in \mathbb{R}$

$\vec{r}\times\vec{b} = \vec{a}\times\vec{b} \Rightarrow \vec{r} = \vec{a} + \lambda'\vec{b}$ where $\lambda' \in \mathbb{R}$

If these two lines intersect, then we must have some values of λ, λ', say λ_0 and λ_0', such that

$$\vec{b} + \lambda_0\vec{a} = \vec{a} + \lambda_0'\vec{b}$$

$$\Rightarrow \quad (1-\lambda_0')\vec{b} + (\lambda_0 - 1)\vec{a} = \vec{0}$$

Since $\vec{a}$ and $\vec{b}$ are non-collinear, we must have $\lambda_0 = \lambda_0' = 1$. The position vector of the point of intersection P can now be evaluated by substituting λ_0 or λ_0' in the corresponding equation:

$$P \equiv \vec{b} + \vec{a} = \vec{a} + \vec{b} \quad \Rightarrow \quad P \equiv 3\hat{i} + \hat{j} - \hat{k}$$

8.(AC) The ellipse and the hyperbola will intersect in four points, and it can be easily deduced that the coordinates of these points will be $x = \pm\frac{3}{\sqrt{10}}$, $y = \pm\frac{1}{\sqrt{5}}$

If the four points are represented by $(x_i, y_i), i = 1, 2, 3, 4,$ we conclude that

$$\sum_{i=1}^{4} x_i = \sum_{i=1}^{4} y_i = 0$$

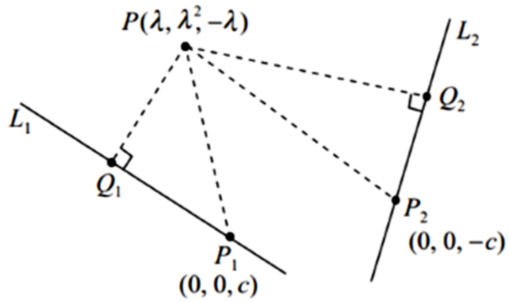
Now, if the variable line is represented by $ax + by + c = 0$, the (algebraic) length of the perpendicular p_i from any one of the four points of intersection is $p_i = \frac{ax_i + by_i + c}{\sqrt{a^2 + b^2}}$

If $\sum_{i=1}^{4} p_i = 0$, we have ; $a\sum_{i=1}^{4} x_i + b\sum_{i=1}^{4} y_i + 4C = 0 \Rightarrow c = 0$

Thus, the variable line always passes through (0, 0)

9.(ABC)

A point point on L_1 is $P_1(0, 0, c)$ and a unit vector along L_1 is $\hat{\mu}_1 = \frac{\hat{i} + m\hat{j}}{\sqrt{1+m^2}}$, while a point on L_2 is $P_2 = (0, 0, -c)$, and a unit vector along L_2 is $\hat{\mu}_2 = \frac{\hat{i} - m\hat{j}}{\sqrt{1+m^2}}$:



Also,

$PP_1^2 = \quad + \lambda^4 + (\lambda + c)^2$

$PP_2^2 = \lambda^2 + \lambda^4 + (\lambda - c)^2$

$P_1Q_1 = |\overrightarrow{P_1P} \cdot \hat{u}_1|$

$P_2Q_2 = |\overrightarrow{P_2P} \cdot \hat{u}_2|$

From the figure and accompanying observations, we can deduce that

$$P_1Q_1 = \frac{\lambda + \lambda^2 m}{\sqrt{1+m^2}}, \; P_2Q_2 = \frac{\lambda - \lambda^2 m}{\sqrt{1+m^2}}$$

If $PQ_1 = PQ_2$ as given in the problem, we have $PP_1^2 - P_1Q_1^2 = PP_2^2 - P_2Q_2^2$

Using the values for these terms and simplifying, we will obtain

$$\lambda = 0, \pm\sqrt{c\left(m + \frac{1}{m}\right)}$$

The correct options are (A), (B) and (C)

10.(BD) Notice that we are required to find the intervals of increase of $f'(x)$ and not $f(x)$. Therefore, we need to first determine $f'(x)$ from $f(x)$, and then check the sign of the derivative of $f'(x)$ in different intervals, i.e., the sign of $f''(x)$. Observe that $f(x)$ is continuous and differentiable at $x = 0$ so that $f'(x)$ is defined at $x = 0$.

Therefore, $f'(x) = \begin{cases} (1+ax)e^{ax}, & x \le 0 \\ 1 + 2ax - 3x^2, & x > 0 \end{cases}$

Notice again that $f'(x)$ is also continuous and differentiable at $x=0$ so that $f''(x)$ is also defined at $x=0$

$$f''(x)=\begin{cases}(2+ax)ae^{ax}, & x\le 0\\ 2a-6x, & x>0\end{cases}$$

Interval(s) of strict increase for $f'(x): f''(x)>0$

$\Rightarrow \quad 2+ax>0$ (if $x\le 0$) and $2a-6x>0$ (if $x>0$)

$\Rightarrow \quad x>\frac{-2}{a}$ (if $x\le 0$) and $x<\frac{a}{3}$ (if $x>0$) $\Rightarrow \frac{-2}{a}<x\le 0$ and $0<x<\frac{a}{3} \Rightarrow \frac{-2}{a}<x<\frac{a}{3}$

And $f'(x)\to$ is continuous at $x=0$

Therefore, $f'(x)$ is strictly increasing on the interval $\left(\frac{-2}{a},\frac{a}{3}\right)$. We see that $L(a)=\frac{a}{3}+\frac{2}{a}$ which gives $L'(a)=\frac{1}{3}-\frac{2}{a^2}$. Thus $L'(3)=\frac{1}{9}$ so that $\frac{1}{L'(3)}=9$

11.(ABD) $\lim\limits_{x\to 0}\frac{f(x)}{\sin^2 x}=\lim\limits_{x\to 0}\frac{\frac{f(x)}{x^2}}{\frac{\sin^2 x}{x^2}}=\lim\limits_{x\to 0}\frac{f(x)}{x^2}=8$ and $\lim\limits_{x\to 0}f(x)=0$

and $\lim\limits_{x\to 0}\frac{g(x)}{2\left(1-\frac{x^2}{2!}+\ldots\right)-x\left(1+x+\frac{x^2}{2!}+\ldots.\right)+x^3+x-2}=\lambda$

$=\lim\limits_{x\to 0}\frac{g(x)}{x^2\left(-2+\frac{x}{2}+\ldots\right)}=\lambda \Rightarrow \lim\limits_{x\to 0}\frac{g(x)}{-2x^2}=\lambda$

$\therefore \quad \lim\limits_{x\to 0}(1+2f(x))^{1/g(x)}=e^{\lim\limits_{x\to 0}\frac{2f(x)/x^2}{g(x)/x^2}}=e^{-8/\lambda}=\frac{1}{e}\Rightarrow \lambda=8$

12.(CD) $(1-x)^n=C_0-C_1x+C_2x^2-C_3x^3+\ldots+(-1)^nC_nx^n$

Multiplying both sides by x^{n-1},

$\Rightarrow \quad x^{n-1}\cdot(1-x)^n=C_0\cdot x^{n-1}-C_1x^n+C_2x^{n+1}-\ldots.+(-1)^nC_n\cdot x^{2n-1}$ ....(1)

Now again multiplying both sides of equation (1) with $(1-x)$,

$\Rightarrow \quad x^{n-1}\cdot(1-x)^{n+1}=(C_0\cdot x^{n-1}-C_1x^n+\ldots.+(-1)^nC_n\cdot x^{2n-1})(1-x)$

Integrating both sides w.r.t. x from 0 to 1, we get

R.H.S.=

$$\int_0^1\left\{C_0(x^{n-1}-x^n)-C_1(x^n-x^{n+1})+C_2(x^{n+1}-x^{n+2})\ldots.\right\}dx$$

$$=C_0\left(\frac{x^n}{n}-\frac{x^{n+1}}{n+1}\right)\Bigg|_0^1-C_1\left(\frac{x^{n+1}}{n+1}-\frac{x^{n+2}}{n+2}\right)\Bigg|_0^1+C_2\left(\frac{x^{n+2}}{n+2}-\frac{x^{n+3}}{n+3}\right)\Bigg|_0^1-\ldots$$

$$=\frac{C_0}{n(n+1)}-\frac{C_1}{(n+1)(n+2)}+\frac{C_2}{(n+2)(n+3)}\ldots\ldots=\int_0^1 x^{n-1}(1-x)^{n+1}dx \qquad =\int_0^1(1-x)^{n-1}\cdot x^{n+1}dx$$

13.(89) Multiply both sides of the equality by cos1°. The general term on the left side can be manipulated as follows:

$$\frac{\cos 1^\circ}{\cos x^\circ \sin(x+1)^\circ}=\frac{\cos((x+1)-x)^\circ}{\cos x^\circ \sin(x+1)^\circ}=\tan x^\circ+\cot(x+1)^\circ$$

Thus, on the left side we will be left with

$$S=(\tan 45^\circ+\cot 46^\circ)+(\tan 47^\circ+\cot 48^\circ)+...+(\tan 133^\circ+\cot 134^\circ)=\tan 45^\circ=1$$

$$\Rightarrow \quad 1=\frac{\cos 1^\circ}{\sin n^\circ}\Rightarrow \sin n^\circ=\cos 1^\circ \Rightarrow n=89$$

14.(3) $\left(f(x+a)-\frac{1}{2}\right)^2=(f(x+a))^2-f(x+a)+\frac{1}{4}=f(x)-(f(x))^2$

Using $x\to x+a$ in the given relation, we have

$$f(x+2a)=\frac{1}{2}-\sqrt{f(x+a)-f((x+a))^2}=\frac{1}{2}-\sqrt{\frac{1}{4}-f(x)+(f(x))^2}=\frac{1}{2}-\left|\frac{1}{2}-f(x)\right|$$

Since $f(x)\in\left[0,\frac{1}{2}\right]$, this implies that $f(x+2a)=f(x)$. Thus, $f(x)$ is periodic with period $2a$.

15.(8) Substituting $x\to(\lambda+(-\lambda)-x)$ or $x\to -x$, we have

$$I=\int_{-\lambda}^{\lambda}\frac{x^2}{(1+\sin^2 x^3)(1+e^{x^7})}dx=\int_{-\lambda}^{\lambda}\frac{x^2}{(1+\sin^2 x^3)(1+e^{-x^7})}dx=\int_{-\lambda}^{\lambda}\frac{x^2e^{x^7}}{(1+\sin^2 x^3)(1+e^{x^7})}dx$$

$$\Rightarrow \quad 2I=\int_{-\lambda}^{\lambda}\frac{x^2}{(1+\sin^2 x^3)}dx$$

Substituting $x^3\to t$, so that $x^2dx=\frac{dt}{3}$, we have

$$I=\frac{1}{2}\cdot\frac{1}{3}\int_{-\lambda^3}^{\lambda^3}\frac{dt}{1+\sin^2 t}=\frac{1}{6}\int_{-\pi/3}^{\pi/3}\frac{\sec^2 t}{1+2\tan^2 t}dt$$

Substituting $\tan t\to y$, we have

$$I=\frac{1}{6}\int_{-\sqrt{3}}^{\sqrt{3}}\frac{dy}{1+2y^2}=\frac{1}{12}\left(\sqrt{2}\tan^{-1}\sqrt{2}\,y\right)\Big|_{-\sqrt{3}}^{\sqrt{3}}=\frac{\sqrt{2}}{6}\tan^{-1}\sqrt{6}$$

16.(104)

$a=52,\ b=51,\ c=1$

P(2 aces are drawn in exactly n draws) = P (exactly 1 ace in $n-1$ draws)

P(second ace in nth draw)

$$=\frac{{}^{48}C_{n-2}\cdot{}^{4}C_1}{{}^{52}C_{n-1}}\times\frac{{}^{3}C_1}{53-n}=\frac{48!\cdot(n-1)!(53-n)!\cdot 4}{(n-2)!(50-n)!\cdot 52!}\times\frac{3}{53-n}$$

$$=\frac{(n-1)(53-n)(52-n)(51-n)\cdot 12}{52\cdot 51\cdot 50\cdot 49}\times\frac{1}{53-n}=\frac{(n-52)(n-51)(n-1)}{13\cdot 17\cdot 50\cdot 49}\equiv\frac{1}{k}(n-a)(n-b)(n-c)$$

$a=52,\ b=51,\ c=1$ and $k=13\cdot 17\cdot 50\cdot 49 \quad \Rightarrow \quad a+b+c=104$

17.(865) $A_1\to 2l-1,\ A_2\to 2m+2,\ A_3\to 2n+3,\ A_4\to 2p$

$\therefore \quad 2l-1+2m+2+2n+3+2p=50$

$\Rightarrow \quad 2l+2m+2n+2p=46 \Rightarrow l+m+n+p=23,\ l,\ m,\ n,\ p \geq 1$

$l'+m'+n'+p'=19,\ l',\ m',\ n',\ p' \geq 0$

$\therefore \quad$ Total number of ways of distribution $= {}^{22}C_3 \Rightarrow p=22$

When A_4 receiving not more than 14 marbles

$l+m+n+p=23$

1 1 1 1

1 1 1 8

$l'+m'+n'+p'=12,\ l',\ m',\ n',\ p' \geq 0$

Number of ways of distribution when A_4 receiving 14 or more marbles $= {}^{15}C_3$

$\therefore \quad$ Number of ways when A_4 receiving not more than 14 marbles $= {}^{22}C_3 - {}^{15}C_3 = 1085 \Rightarrow q = 1085$

18.(191)

The number of solutions of $x_1 + x_2 + \ldots x_k = n$, under given conditions

= coefficient of x^n in $(x+x^2+x^3+\ldots\ldots)(x^2+x^3+\ldots\ldots\ldots)\ldots\ldots\ldots(x^k+x^{k+1}+\ldots\ldots\ldots)$

= coefficient of x^n in $x^{1+2+\ldots\ldots\ldots+k}(1+x+x^2+\ldots\ldots\ldots)^k$ = coefficient of x^n in $x^{\frac{k(k+1)}{2}}(1-x)^{-k}$

= coefficient of x^{n-r} in $(1-x)^{-k}$ $\quad$ [Assuming $\frac{k(k+1)}{2}=r$]

= coefficient of x^{n-r} in $[1+{}^kC_1 x + {}^{k+1}C_2 x^2 + \ldots\ldots\ldots]$

${}^{k-1+n-r}C_{n-r} = {}^{k-1+n-r}C_{k-1}$ where $r = \frac{k(k+1)}{2}$

Now, $k-1+n-r = k-1+n-\frac{k(k+1)}{2} = \frac{1}{2}(2n-k^2+k-2)$

Hence, required number of solutions

$= {}^mC_{n-r} = {}^mC_{k-1}$, where $m = \frac{1}{2}[2n-k^2+k-2]$